
MATH-232 — Proba-Stat

Examen final 2021

F.Dumoncel – Février 2022

Question 1 :

We have 10 **blue** card numbered from 1 to 10 and 5 **green** card numbered from 1 to 5. How many different permutations of the card can you obtain

- (a) if you can see the colors and the numbers ?
 - (b) if you can see the colors but not the numbers ?
 - (c) if you can see the numbers but not the colors ?
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Question 2 :

Assume that a person gets the C -virus when exposed to it with probability

- 0.01 if the person is vaccinated and wearing a mask,
- 0.05 if the person is vaccinated and not wearing a mask,
- 0.1 if the person is not vaccinated and wearing a mask,
- 0.5 if the person is not vaccinated and not wearing a mask.

Assume that a person wears a mask with probability $\frac{1}{2}$ and that a person is vaccinated with a probability $\frac{1}{3}$. Furthermore, assume that people wear masks independently of being vaccinated. What is the probability that a person was vaccinated, given that she/he wore a mask and got the C -virus when exposed to it ?

Question 3 :

Let Θ be a random variable uniformly distributed over $[-\pi, \pi]$. This means that the density function of Θ equals $\frac{1}{2\pi}$ on $[-\pi, \pi]$ and 0 otherwise. Let $X = \cos(\Theta)$ and $Y = \sin(\Theta)$.

- (a) Are X and Y uncorrelated ?
 - (b) Are X and Y independent ?
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Question 4 :

Assume that in a class of n students, any combination of 3 students forms a friend-triplet with probability e^{-n} . Show that the number of friend-triplet in a class of n students is greater of equal to 1 with probability going to 0 as $n \rightarrow \infty$. *Hint : You can use Markov's inequality to approach this problem.*

Question 5 :

Let X_1, X_2 be i.i.d $\mathcal{N}(0, 1)$ and let $Y_1 = X_1 + X_2$ and $Y_2 = X_1 - X_2$.

- (a) What is the joint distribution of (Y_1, Y_2) ?
 - (b) Are Y_1 and Y_2 independent ?
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Question 6 :

Let Y_1, \dots, Y_n be i.i.d such that Y_i is uniformly distributed on $[a, b]$ where $a < b$. This means that the density function of Y_i equals $\frac{1}{b-a}$ on $[a, b]$ and 0 otherwise. Find estimators for a and b using the method of moments. *Hint* : $(x^3 - y^3) = (x - y)(x^2 + xy + y^2)$.

Question 7 :

Let Y_1, \dots, Y_n be i.i.d such that Y_i is uniformly distributed on $[0, \theta]$.

- (a) Show that $\max(Y_1, \dots, Y_n)/\theta$ is a pivot.
 - (b) Using this pivot, give an equi-tailed two sided confidence interval for θ with confidence level $1 - \alpha = \frac{7}{8}$ (i.e $\alpha = \frac{1}{8}$) in the case $n = 2$ and $\max(Y_1, Y_2) = 1$.
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Question 8 :

Consider the binary hypothesis test where a single random variable Y is observed. Under H_0 , the random variable is uniformly distributed over $[0, 3]$, and under H_1 it is uniformly distributed over $[1, 4]$.

- (a) Give a test (i.e define the set of values of Y for which you declare H_0 or H_1) such that the sum of the false positive and false negative probabilities is minimized.
- (b) Give a test such that either the false positive or the false negative probability is zero and the other of the two probabilities is minimized.